

Numerical Study on the Magnetic Fluid Delivering Through a non-Newtonian Viscoplastic Blood

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Abstract— Nowadays the application of magnetic fields in medical problems finds a special place and attracts the attention of many investigations. The purpose of current study is to show the usage of magnetic field in targeting drug particles through a viscoplastic blood flow. The governing non-linear fluid equations coupled with the magnetic fields and solved numerically by pseudo-transient SIMPLE algorithm. The simulation results show the eddy formation in the vicinity of magnetic source increases the drug absorption probability on the target zone. The central line velocity profile and lower wall shear stress distribution are also compared for both the Casson and Newtonian models. It was found that under the action of external magnetic field, higher viscosity estimation in the Casson model creates 35.38% higher shear force than the Newtonian condition.

Index Terms— Biofluid, Blood, Fluid Mechanics, Magnetic Drug Targeting, Magnetic Fields, Straight Artery, Viscoplastic Fluid.

1 INTRODUCTION

Magnetic drug targeting (MDT) is one of important and relatively new application of magnetic fields in vivo environment [1, 2]. In this method, the drug elements is magnetized and then injected to the blood flow as a dilution. One magnetic field is placed near the tumour or target region. The drug particles move through the blood flow till they reach the magnetic source, they absorb by the magnetic field and shift toward the target. With this technique the efficiency of drug absorption increases and the other cells of body won not be affected [3]. Nowadays many researches have been done in this field, between the years 1996-1999 Y. Haik and et.al suggested Biomagnetic Fluid Dynamics (BFD) mathematical formulation for the first time [4, 5]. Later in 2003 and 2005 E. E. Tzirtzilak and et. al extended the formulation [6, 7]. In these papers, they simulated Newtonian blood flow in 2D and 3D channel and analysed the effect of magnetic dipole, electrical wire and uniform magnetic fields on the blood fluid field. Alimohamadi and et.al studies the usage of external magnetic field in different kinds of aneurysm vessels modelling by commercial software COMSOL [8, 9]. The effect of vessel porosity and transient solution on the drug accumulation around the diseased zone were investigated by Alimohamadi and et.al. The Numerical simulation of biomagnetic blood flow in stenosis channel under the influence of eternal magnetic field is brought up in paper [10]. However, in this study the arterial walls assumed rigid and non-Newtonian viscosity of blood was neglected. In paper [11] an experimental test of magnetic drug steering to the mouses' liver. In a recent research in 2014 by Alimohamadi and et.al biomagnetic blood flow in a constricted artery is simulated numerically [12]. The atherosclerotic plaque and vessel walls were considered as porous medium in this article and non-Newtonian behaviour of blood was taken to account by Power Law model. They results showed by applying strong magnetic field two eddies formed on the plaque edges which not only heated the fatty deposits but also exposed them to high shear force.

Among works in this area, there are few studies of investigation of magnetic drug delivery in viscoplastic blood flow. The main purpose of simulation study is to compare magnetic drug targeting in two viscoplastic and Newtonian blood flow.

In this study Casson formulation is used as a viscoplastic model and coupled fluid flow and magnetic equations solved numerically.

2 MODEL EQUATIONS

2.1 Governing Equations

The blood flow inside the vessel is governed by extended Cauchy equations under the action of external magnetic field. These equations include continuity and momentum in x and y direction as follows:

$$\iint_A \mathbf{V} \cdot d\mathbf{A} = 0 \quad (1)$$

$$\iint_A \rho u \mathbf{V} \cdot d\mathbf{A} = - \iint_A P d\mathbf{A} + \iint_A \tau d\mathbf{A} + \iiint_V F_x dV \quad (2)$$

$$\iint_A \rho v \mathbf{V} \cdot d\mathbf{A} = - \iint_A P d\mathbf{A} + \iint_A \tau d\mathbf{A} + \iiint_V F_y dV \quad (3)$$

Where \mathbf{V} is two dimensional velocity in x and y direction, \mathbf{n} is the normal vector to the surface, P is the pressure and $\rho=1050$ kg/m³ is human blood density [13]. In 1987, Papanastasiou [14] suggested a mathematical formulation for modelling viscoplastic rheology of blood. According to this model shear-stress tensor is related to shear rate as:

$$\bar{\tau} = \mu \bar{\gamma} \quad (4)$$

$$\mu = \left[\sqrt{\mu_\infty} + \sqrt{\frac{\tau_y}{|\bar{\gamma}|}} (1 - e^{-\sqrt{m|\bar{\gamma}|}}) \right]^2$$

In (4) $\tau_y = 0.01082$ Pa and $\mu_\infty = 0.0031$ Pa.s are yield stress and infinite shear viscosity respectively [15] and it is shown [16], for $m>100$ this model has a good match with Casson equation. In Fig 1, the blood viscosity of Casson model is compared with Newtonian assumption. As seen, for low shear rate, Casson model predicts about 2 order of time higher viscosity than Newtonian case. However, in high shear rate, two models overlap and assign same value to the blood viscosity.

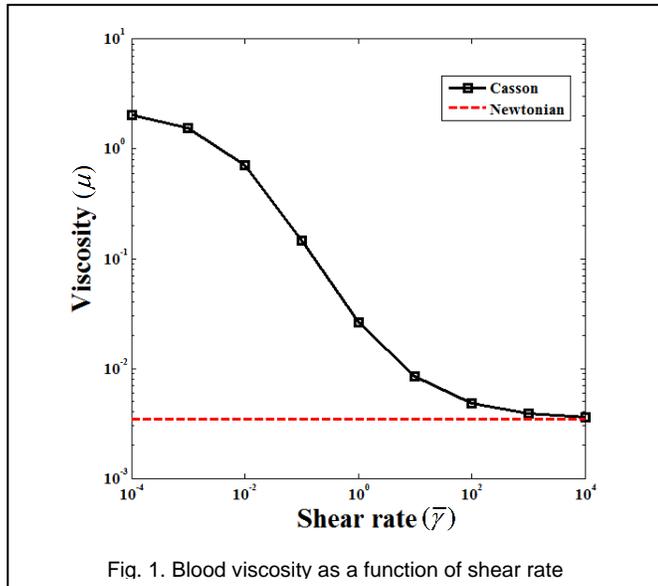


Fig. 1. Blood viscosity as a function of shear rate

In (2) and (3), F_x and F_y represent magnetic forces in x and y directions which is defined as [6]:

$$F_x = \mu_0 \chi H \frac{\partial H}{\partial x}$$

$$F_y = \mu_0 \chi H \frac{\partial H}{\partial y}$$

(5)

χ and μ_0 are constant magnetic susceptibility and permeability of vacuum. H is the magnetic field intensity which depends on the type of magnetic source.

2.2 Geometry and Parameters

The blood flows inside a two dimensional human abdominal straight artery. The flow is assumed homogenous, laminar, steady state, viscoplastic and in our simulation, the fluid surface interaction (FSI) is negligible compared to other parameters [17]. A permanent magnetic dipole is embedded below the lower vessel wall. The schematic geometry of simulation domain is presented in Fig 2.

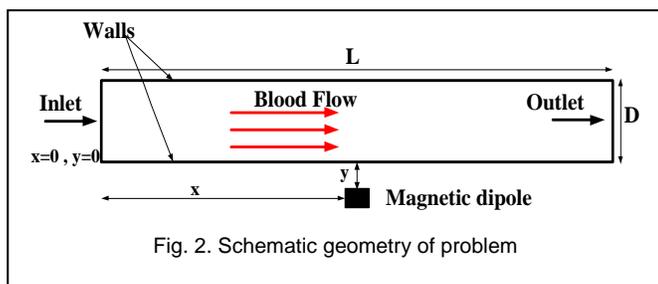


Fig. 2. Schematic geometry of problem

For non-dimensionalization we used mean inlet velocity (U_∞), vessel diameter (D) and (H_0) is the magnetic field strength at the point of magnetic source. Based on this assumption two dimensionless parameters Reynold (Re), Bingham (Bi) and Magnetic (Mn) numbers are calculated by:

$$Re = \frac{\rho U_\infty D}{\mu_\infty}, \quad Bi = \frac{\tau_y D}{\mu_\infty U_\infty}, \quad Mn = \frac{\mu_0 \chi H_0^2}{\rho U_\infty^2}$$

To obtain numerical results, the following dimensionless boundary conditions are required:

Inlet: $(x = 0, 0 \leq y \leq 1) : u(y) = 4(y - y^2), v = 0$

Out let: $(x = 10, 0 \leq y \leq 1) : \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0$

Upper wall: $(y = 1, 0 \leq x \leq 10) : u = v = 0$

Lower wall: $(y = 0, 0 \leq x \leq 10) : u = v = 0$

3 NUMERICAL PROCEDURE

The coupled velocity-pressure equations (1)-(3) have discretised based on control volume method. A general C++ code has been developed for solving trial and error loops. The convective and diffusive terms are calculated with second order upwind and because of instability of solution, pseudo-transient SIMPLE algorithm is used.

D.N ku [19] represented the physiological blood flow profile for straight abdominal aorta. According to this test, Reynolds number can be considered 300 averagely. Two other appeared dimensionless numbers Bi and Mn are chosen 0.136 and 100 respectively.

4 RESULTS

For analysing the effect of external magnetic field on the blood flow, the results for two Casson and Newtonian models are presented and compared with each other.

Fig 3 shows velocity contour and streamlines for a) Newtonian and b) Casson models. In the vicinity of magnetic source due to high magnetic gradient, one big eddy is formed and stagnate flow above the vessel surface. It can be seen, Casson model captured smaller eddy than Newtonian case because in low shear rate the Casson model predicts higher viscosity for blood (look Fig 1). Infact, powerful inertia forces in the model weaken the effect of magnetic drag and the flow passes the magnetic zone without any deviation.

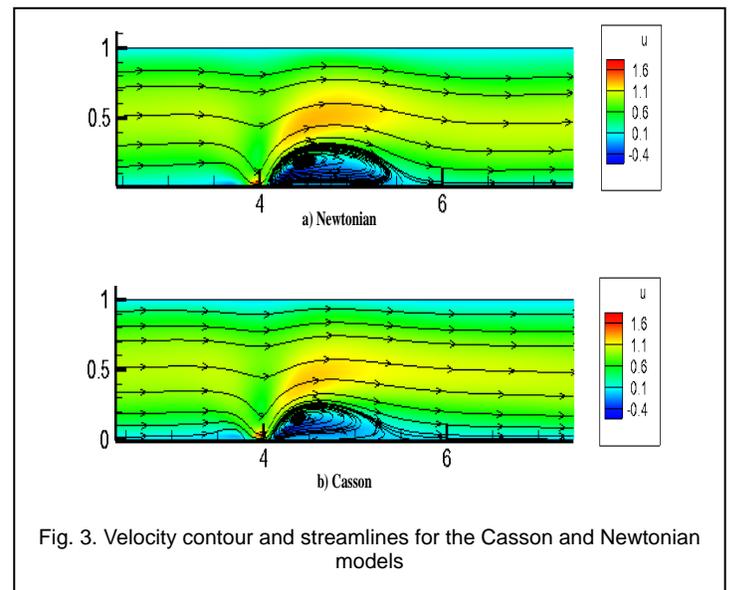


Fig. 3. Velocity contour and streamlines for the Casson and Newtonian models

Non-dimensional velocity magnitude for both the Casson and Newtonian blood flow in the middle of artery is depicted in Fig 4. As shown both models have a same trend, velocity magnitude goes down and then up in neighbourhood of magnetic source. This is because of flow recirculation which acts as solid body. Before eddy magnetic drag decelerates flow but above it narrowing the available cross sectional area provides a nozzle condition inside the vessel and pushes the flow forward. It can also concluded from figure that in Newtonian model bigger vortex occupies larger area and flow moves with 5.7% higher speed than the Casson model.

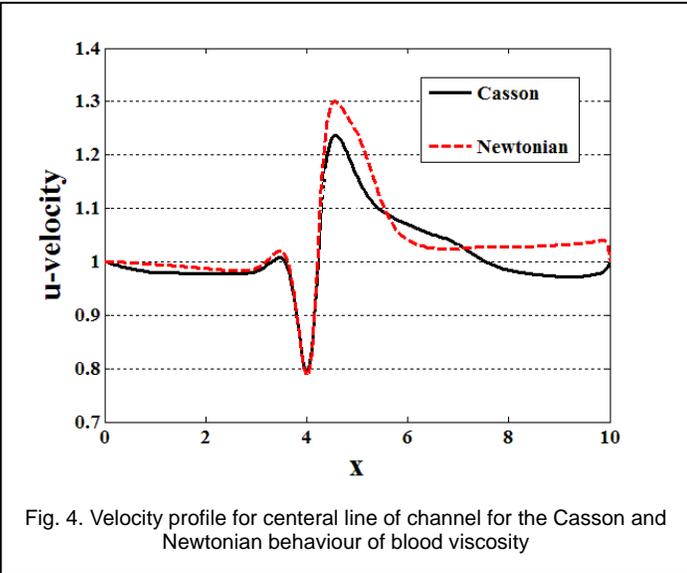


Fig. 4. Velocity profile for central line of channel for the Casson and Newtonian behaviour of blood viscosity

In Fig 5 shear stress distribution on the lower vessel wall for two different viscosity model is illustrated. Shear stress value relates to the viscosity directly therefore, the slight difference in value and shape of graph can be referred to the different viscosity functions. More viscous flow in the Casson model sticks to the wall and exerts higher shear stress to the surface rather Newtonian case. In addition, by advent of counterclockwise vortex on the lower wall, blood flow direction had been reversed and negative shear stress is appeared throughout the magnetic zone.

Shear force is one of important fluid mechanics parameter which is defines as the integration of shear stress along the surface:

$$F_{shear} = \iint_s \tau_w ds \quad (6)$$

Fig 6 depicts the lower wall shear force magnitude of various magnetic field strength for the Casson and Newtonian models. With strengthening magnetic field a bigger eddy is created and the negative shear stress scatters is wider area. So, as this figure shows with doubling magnetic number from Mn=50 to Mn=100, the shear force magnitude for Newtonian and the Casson cases declines 19.23% and 23.63% respectively. Near eddy shear rate is too small and in this condition viscous Casson fluid resists again flowing. As a result, it is justifiable that for both magnetic number, the Casson model estimates higher shear force respect to the Newtonian.

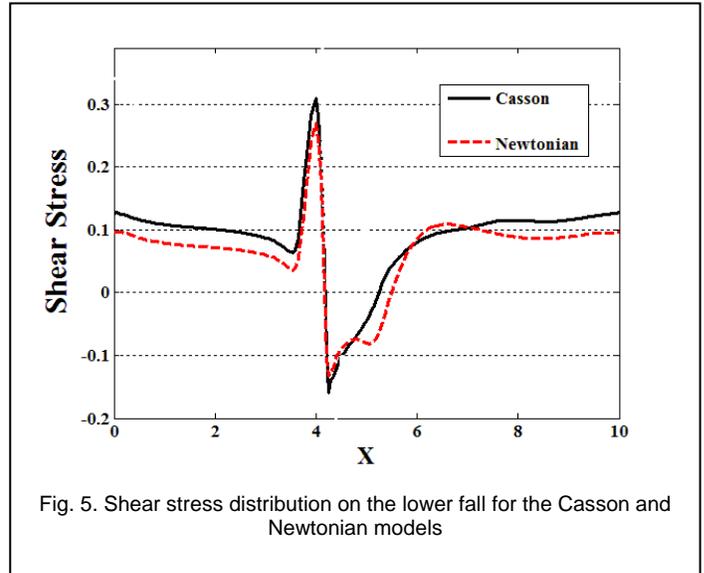


Fig. 5. Shear stress distribution on the lower fall for the Casson and Newtonian models

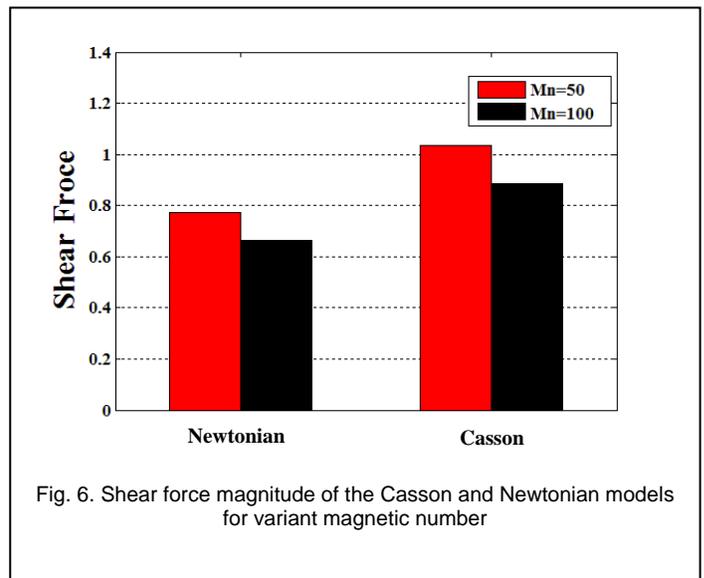


Fig. 6. Shear force magnitude of the Casson and Newtonian models for variant magnetic number

Maximum viscosity and pressure value for the Newtonian and Casson models under the action of two different magnetic field numbers are reported in Table 1. As seen, with increasing magnetic number the maximum pressure of flow goes up. In view of the fact that velocity and pressure have opposite relation, so with slowing down flow by stronger magnetic force (doubling magnetic number) the maximum pressure rises about 2 times for each model. As was mentioned before, the Casson model assigns higher viscosity to the blood flow as far as in Mn=50 and Mn=100 maximum predicted viscosity of Casson model is 27.53X and 8.40X larger than the Newtonian.

TABLE 1
MAXIMUM VISCOSITY AND PRESSURE OF THE NEWTONIAN AND CASSON MODELS FOR DIFFERENT MAGNETIC NUMBER

| Magnetic intensity | Mn=50 | | Mn=100 | |
|--------------------|--------------|--------|--------------|--------|
| | η_{max} | Pmax | η_{max} | Pmax |
| Newtonian | 0.00345 | 377.00 | 0.00345 | 754.38 |
| Casson | 0.095 | 358.37 | 0.029 | 750.52 |

5 CONCLUSION

In this paper we mainly studied magnetic drug targeting (MDT) in a two dimensional straight artery. We compared the size of eddy, velocity contour and shear force magnitude for two different viscosity behaviour the Casson and Newtonian models. The results show under same magnetic field strength the Casson flow stands out against flow separation harder than the Newtonian case. Moreover, higher viscosity of the Casson model in low shear rate results in higher shear stress and shear force value along the artery surface.

6 REFERENCES

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